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Replica-symmetric solutions of a dilute Ising ferromagnet in a random field

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Abstract. We use the replica method in order to obtain an expression for the variational free energy of an Ising ferromagnet on a Viana-Bray lattice in the presence of random external fields. Introducing a global order parameter, in the replica-symmetric context, the problem is reduced to the analysis of the solutions of a nonlinear integral equation. At zero temperature, and under some restrictions on the form of the random fields, we are able to perform a detailed analysis of stability of the replica-symmetric solutions. In contrast to the behaviour of the Sherrington-Kirkpatrick model for a spin glass in a uniform field, the paramagnetic solution is fully stable in a sufficiently large random field.

PACS. 75.10.Nr Spin-glass and other random models – 89.75.-k Complex systems

1 Introduction

Magnetic systems with quenched disorder, including spin glasses and ferromagnets in a random field, have been intensively studied during the last decades [1]. There are many applications of these disordered systems, ranging from the study of the behavior of random magnets, which is a traditional ground test for the ideas of statistical mechanics, to the analysis of different sorts of optimization problems in distinct areas of science. The mean-field version of an Ising spin glass, with Gaussian distribution of exchange interactions, also known as the Sherrington-Kirkpatrick model, which can be solved by the replica method, displays a low-temperature glassy phase, characterized by the instability of a replica-symmetric solution, which indicates the need of breaking replica symmetry and the existence of many ultrametrically organized states. In contrast to this rich behavior, the mean-field, Curie-Weiss, version of an Ising ferromagnet in a random field (RFIM), which can be solved without recourse to the replica method, leads to rather uninteresting, replica-symmetric, exact solutions. There have been, however, some indications that a ferromagnet in a random field may have a glassy behavior.

We were motivated to look again at this problem by a number of early and some more recent investigations of the RFIM. De Almeida and Bruinsma [2] have done some calculations, beyond the usual Curie-Weiss, mean-field, approximation, for analyzing the behavior of a bond-diluted Ising antiferromagnet in a field. For large dimensionality, these calculations lead to the presence of a glassy region in the applied field versus temperature phase diagram, between paramagnetic and ordered phases, which can be shown to disappear in the limit of infinite dimension. On the basis of the equivalence, at a mean-field level, between the critical behavior of a ferromagnet in a random field and of a dilute antiferromagnet in a uniform field, this result gives an indication of the possible existence of a glassy phase in the RFIM. A glassy behavior is also present in a recent "extended mean-field" calculation by Pastor and collaborators [3] for the phase diagram of the RFIM. These results are claimed to agree with earlier work of Mézard and Young [4] using a screening approximation in order to characterize the instability of the replica-symmetric solutions, to lowest order in $1/m$, in a renormalization-group calculation for an m-component spin ferromagnet in a random field. It should mentioned that calculations for the RFIM on a Bethe lattice already indicate a rich ground-state structure [5] and peculiar hysteresis effects [6]. Also, field-theoretical renormalization-group calculations for a soft-spin version of the RFIM, which were confirmed by a formulation of the dynamics, have shown the need to include extra terms involving replicas with different indices, which in turn may lead to an instability of the replica-symmetric solution in the paramagnetic region [7]. The prediction of a glassy phase in the calculations beyond the usual mean-field approximation were the main motivation to revisit this problem. We then decided to use a model devised by Viana and Bray [8], which is designed to gauge the effects of the (finite) connectivity of a lattice.

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According to the original work of Viana and Bray, we consider an Ising model with pair interactions J_{ij}, between all sites i and j, such that $J_{ij} = J > 0$, with probability c/N, where N is the total number of sites, and $J_{ij} = 0$, with probability $1 - c/N$. This choice of interactions gives rise to the so-called Viana-Bray lattice. The parameter $c > 0$ can be regarded as the (finite) mean connectivity per site. The solutions of an Ising spin-glass on the Viana-Bray lattice have been first analyzed in the vicinity of the transition temperature [8]. The more involved low-temperature behavior has been considered by Kanter and Sompolinsky [9], and Mézard and Parisi [10]. The analysis of stability of the replica-symmetric solutions near percolation ($c = 1$) has been carried out by de Dominicis and collaborators [11]. Although there are calculations for the RFIM on a Bethe lattice, there is no comprehensive analysis on a graph as the Viana-Bray lattice. According to the previous work for the Ising spin glass, we introduce a global order parameter and formulate the replica-symmetric solutions for this problem in terms of an integral equation. In the ground state, and under some conditions on the form of the random fields, we show that the replica-symmetric solutions can be written as a series expansion. We have been able to perform a detailed analysis of stability of this replica-symmetric solution. In a sufficiently strong random field, the explicit calculation of the eigenvalues of a functional Hessian form shows the stability of the paramagnetic solution, which seems to preclude the existence of a glassy phase (in contrast to earlier expectations and the results of Pastor and collaborators [3]). Some numerical calculations confirm these findings for the paramagnetic solution, and indicate that the replica-symmetric ferromagnetic solution is also stable in the ferromagnetic region of the phase diagram.

The layout of this paper is as follows. In Section 2, we define the model and formulate the replica-symmetric solutions in terms of an integral equation for the global order parameter. The analysis of stability of the paramagnetic solution is reported in Section 3. Some conclusions, and connections with recent work, are presented in Section 4.

2 Formulation of the problem

The ferromagnetic Ising model on the Viana-Bray lattice is given by the Hamiltonian

$$H = -\sum_{(ij)} J_{ij}\sigma_i\sigma_j - \sum_{i=1}^{N} H_i\sigma_i, \tag{1}$$

where (ij) refers to a pair of sites, $\sigma_i = \pm 1$ for all sites, and $\{J_{ij}\}$ and $\{H_i\}$ are sets of independent, identically distributed, random variables, associated with probability distributions

$$p_J(J_{ij}) = \frac{c}{N}\delta(J_{ij} - J) + \left(1 - \frac{c}{N}\right)\delta(J_{ij}), \tag{2}$$

and

$$p_H(H_i) = \frac{1}{2}\delta(H_i - H_R) + \frac{1}{2}\delta(H_i + H_R), \tag{3}$$

where J, c, and H_R are positive parameters. This distribution of exchange interactions is supposed to mimic a lattice of mean finite connectivity c.

Using the replica method, it is not difficult to write the variational free energy [11]

$$f = \frac{1}{\beta}\lim_{n\to 0}\frac{1}{n}\Bigg\{\frac{c}{2} + \frac{c}{2}\sum_{r=0}^{n}\sum_{(\alpha_1,...,\alpha_r)} b_r q^2_{\alpha_1,...,\alpha_r} - \ln\int_{-\infty}^{+\infty} dH p_H(H)\,\mathrm{Tr}_\sigma \exp\left[G(\{\sigma_\alpha\}) + \beta H\sum_{\alpha=1}^{n}\sigma_\alpha\right]\Bigg\}, \tag{4}$$

where $\beta = 1/(k_B T)$, T is the temperature, n is the number of replicas,

$$b_r = \cosh^n(\beta J)\tanh^r(\beta J), \tag{5}$$

and the trace is taken over the set of replica spin variables $\{\sigma_\alpha\}$. The parameter $q_{\alpha_1,...,\alpha_r}$ is the expected value of the product $\sigma_{\alpha_1}...\sigma_{\alpha_r}$. The global order parameter $G(\{\sigma_\alpha\})$ is defined as

$$G(\{\sigma_\alpha\}) = c\sum_{r=0}^{n}\sum_{(\alpha_1,...,\alpha_r)} b_r q_{\alpha_1,...,\alpha_r}\sigma_{\alpha_1}...\sigma_{\alpha_r}. \tag{6}$$

In the $n \to 0$ limit, the minimization of this variational free energy with respect to the set of variables $\{q_{\alpha_1,...,\alpha_r}\}$ leads to the stationary conditions

$$q_{\alpha_1,...,\alpha_r} = \frac{1}{Z_G}\int_{-\infty}^{+\infty} dH p_H(H)\,\mathrm{Tr}_\sigma\sigma_{\alpha_1}...\sigma_{\alpha_r} \times\exp\left[G(\{\sigma_\alpha\}) + \beta H\sum_{\alpha=1}^{n}\sigma_\alpha\right], \tag{7}$$

where

$$Z_G = \int_{-\infty}^{+\infty} dH p_H(H)\,\mathrm{Tr}_\sigma \exp\left[G(\{\sigma_\alpha\}) + \beta H\sum_{\alpha=1}^{n}\sigma_\alpha\right]. \tag{8}$$

In the context of the replica-symmetric Ansatz, we define an effective field h, associated with an effective probability distribution $p(h)$, which is equally applied on all of the replica spin variables. We then write

$$q_{\alpha_1,...,\alpha_r} = \int_{-\infty}^{+\infty} dh p(h)\tanh^r(\beta J), \tag{9}$$

from which we have

$$p(h) = \int_{-\infty}^{+\infty} dH p_H(H)\int_{-\infty}^{+\infty}\frac{dy}{2\pi} \times\exp\left[-iy(h - H) + G\left(\frac{iy}{\beta}\right) - c\right]. \tag{10}$$

Depending on the quantity of interest, it may be more convenient to work either with the global order parameter

G or with the effective probability distribution p. In the replica-symmetric context, it is easy to see that

$$G(\{\sigma_\alpha\}) = G\left(\sum_{\alpha=1}^{n} \sigma_\alpha\right). \quad (11)$$

Therefore, taking into account the double-delta distribution (3), the extremization of the variational free energy is reduced to the problem of searching the solutions of the non-linear integral equation

$$p(h) = \int_{-\infty}^{+\infty} \frac{dy}{2\pi} \times \exp\left[-iyh + \ln\cosh(iyH_R) - c + G\left(\frac{iy}{\beta}\right)\right], \quad (12)$$

where

$$G(y) = c\int_{-\infty}^{+\infty} dx p(x) \times \exp\left\{y \tanh^{-1}\left[\tanh(\beta J)\tanh(\beta x)\right]\right\}. \quad (13)$$

In the context of the replica-symmetric Ansatz, it is known that the Viana-Bray spin-glass problem, with a symmetric distribution of exchange interactions and no external fields, can also be formulated in terms of a similar integral equation for the distribution of the effective fields [9,10]. In this spin-glass case, if we restrict to the analysis of the ground state ($\beta \to \infty$), it is possible to write an analytic solution as a sum of delta functions peaked at integer multiples of the variance J of the distribution of exchanges. In the present case, however, a similar solution requires the additional assumption that H_R/J is restricted to the set of integer numbers. Under these conditions, in the ground state, the integral equation (12) can be exactly solved in terms of sums of delta functions.

In the ground state ($\beta \to \infty$), it is possible to show that

$$G\left(\frac{iy}{\beta}\right) = c\int_{-\infty}^{-J} dx p(x)\exp(-iyJ) + c\int_{-J}^{+J} dx p(x)\exp(ixy) + c\int_{+J}^{\infty} dx p(x)\exp(iyJ). \quad (14)$$

Assuming that $H_R = \omega J$, with $\omega = 0, 1, 2, ...$, we can also write

$$G\left(\frac{iy}{\beta}\right) = A + B\exp(iyJ) + C\exp(-iyJ), \quad (15)$$

with

$$c = A + B + C, \quad (16)$$

$$\frac{1}{c}A = \frac{1}{2}e^{(A-c)}\left[\left(\frac{C}{B}\right)^{\omega/2} + \left(\frac{C}{B}\right)^{-\omega/2}\right] I_\omega\left(2\sqrt{BC}\right), \quad (17)$$

and

$$\frac{1}{c}B = 1 - \frac{C}{2}e^{(A-c)}\int_0^1 dt e^{(tC)}\left\{\left[\frac{C(1-t)}{B}\right]^{\frac{\omega-1}{2}} \times I_{\omega-1}\left(2\sqrt{BC(1-t)}\right) + \left[\frac{C(1-t)}{B}\right]^{-\frac{\omega+1}{2}} \times I_{\omega+1}\left(2\sqrt{BC(1-t)}\right)\right\}, \quad (18)$$

for $w \geq 1$, where $I_\omega(x)$ is the modified Bessel function. Both ferromagnetic ($B \neq C$) and paramagnetic ($B = C$) solutions are represented by this expression. The effective probability distribution p is written as a sum of delta functions,

$$p(h) = \sum_{k=-\infty}^{+\infty} a_k \delta(h - kJ), \quad (19)$$

where

$$a_k = \frac{1}{2}\exp(A-c) \times \left[\left(\frac{B}{C}\right)^{\frac{k+\omega}{2}} I_{k+\omega}\left(2\sqrt{BC}\right) + \left(\frac{B}{C}\right)^{\frac{k-\omega}{2}} I_{k-\omega}\left(2\sqrt{BC}\right)\right]. \quad (20)$$

3 Analysis of stability

The analysis of stability of the replica-symmetric solutions is based on the investigation of the eigenvalues of the Hessian matrix associated with the variational free energy, given by equation (4), which can be rewritten as

$$\beta f[G] - \frac{c}{2} = \sum_{r=0}^{n} \sum_{(\alpha_1,...,\alpha_r)} \frac{2^{-2n}}{2cb_r}\left[\mathrm{Tr}_\sigma \sigma_{\alpha_1}...\sigma_{\alpha_r} G(\{\sigma_\alpha\})\right]^2 - \ln \mathrm{Tr}_\sigma \exp\left[G(\{\sigma_\alpha\}) + \ln\cosh\left(\beta H_R \sum_{\alpha=1}^{n}\sigma_\alpha\right)\right]. \quad (21)$$

We then write

$$\mathrm{Tr}_\tau \frac{\delta^2 \beta f[G]}{\delta G(\{\sigma_\alpha\})\,\delta G(\{\tau_\alpha\})}\varphi(\{\tau_\alpha\}) = \lambda\varphi(\{\sigma_\alpha\}), \quad (22)$$

which can be cast in the form

$$\varphi(\{s_\alpha\}) = c\lambda \mathrm{Tr}_\tau \exp\left[\beta J\sum_{\alpha=1}^{n}\tau_\alpha s_\alpha\right]\varphi(\{\tau_\alpha\}) + \frac{c}{Z_G}\mathrm{Tr}_\tau \times \exp\left[G(\widehat{\tau}) + \ln\cosh(\beta H_R \widehat{\tau}) + \beta J\sum_{\alpha=1}^{n}\tau_\alpha s_\alpha\right]\varphi(\{\tau_\alpha\}) - \frac{1}{Z_G}G(\widehat{s})\,\mathrm{Tr}_\tau \exp\left[G(\widehat{\tau}) + \ln\cosh(\beta H_R\widehat{\tau})\right]\varphi(\{\tau_\alpha\}), \quad (23)$$

where

$$\widehat{\tau} = \sum_{\alpha=1}^{n} \tau_\alpha, \qquad \widehat{s} = \sum_{\alpha=1}^{n} s_\alpha. \tag{24}$$

In the $n \to 0$ limit, it is easy to show that there is a constant eigenvector, $\varphi(\{s_\alpha\}) = \text{constant}$, with $1/c$ as the associated eigenvalue.

According to the work of De Dominicis and Mottishaw [11], in the context of the replica-symmetric approximation the space of 2^n eigenvectors is spanned by a set of eigenvectors parameterized by functions of two variables, of the form

$$\varphi(\{\sigma_\alpha\}) = \varphi_{\{\mu_\alpha\}}(\widehat{\sigma}; q_{\sigma\mu}), \tag{25}$$

where

$$\widehat{\sigma} = \sum_{\alpha=1}^{n} \sigma_\alpha, \qquad q_{\sigma\mu} = \sum_{\alpha=1}^{n} \sigma_\alpha \mu_\alpha, \tag{26}$$

and the spin configuration $\{\mu_\alpha\}$ is used to label the eigenvectors. From equation (23), in the $n \to 0$ limit, we derive an integral equation for the eigenvalues,

$$\begin{aligned}
\varphi_\mu(x,y) = &-\frac{G(x)}{\exp(c)} \iint_{-\infty}^{+\infty} dm dr \iint_{-\infty}^{+\infty} \frac{du dv}{(2\pi)^2} \\
&\times \left[\frac{\cosh(\beta u + \beta v)}{\cosh(\beta u - \beta v)}\right]^{\mu/2} \exp\left[G\left(\frac{im}{\beta}\right) - imu - irv \right.\\
&\left. + \ln\cosh(iH_R m)\right] \varphi_\mu\left(\frac{im}{\beta}, \frac{ir}{\beta}\right) \\
&+ c \iint_{-\infty}^{+\infty} dm dr \iint_{-\infty}^{+\infty} \frac{du dv}{(2\pi)^2} \\
&\times \left\{\lambda + \exp\left[G\left(\frac{im}{\beta}\right) + \ln\cosh(iH_R m) - c\right]\right\} \\
&\times \exp(imu + irv) \left[\frac{\cosh(\beta J + \beta u + \beta v)}{\cosh(\beta J - \beta u - \beta v)}\right]^{\frac{x+y}{4}} \\
&\times [\cosh(\beta J + \beta u + \beta v)\cosh(\beta J - \beta u - \beta v)]^{\frac{\mu}{4}} \\
&\times \left[\frac{\cosh(\beta J + \beta u - \beta v)}{\cosh(\beta J - \beta u + \beta v)}\right]^{\frac{x-y}{4}} \\
&\times [\cosh(\beta J + \beta u - \beta v)\cosh(\beta J - \beta u + \beta v)]^{-\frac{\mu}{4}} \\
&\times \varphi_\mu\left(\frac{im}{\beta}, \frac{ir}{\beta}\right).
\end{aligned} \tag{27}$$

First, we find the longitudinal eigenvalues, in other words, the eigenvalues in the subspace spanned by $\varphi_{\{\mu_\alpha\}}(\widehat{\sigma}; q_{\sigma\mu}) = \varphi(\widehat{\sigma})$. In the ground state, it is not difficult to see that these longitudinal eigenvectors can be written as

$$\varphi_L\left(\frac{ix}{\beta}\right) = A_L + B_L \exp(ixJ) + C_L \exp(-ixJ). \tag{28}$$

The problem reduces to the calculation of the eigenvalues of a 3×3 matrix, which are given by $\lambda_1 = 1/c$, associated with the constant eigenvector, and

$$\lambda_{2,3} = \frac{1}{c}(1 - A) \pm D, \tag{29}$$

where $A = ca_0$ is given by equation (17), and $D = (a_1 a_{-1})^{1/2}$ is given by the expression

$$\begin{aligned}
D = \frac{1}{2}\exp(A - c) \Bigg\{ &\left(\frac{C}{B}\right)^{\omega} I_{\omega+1}\left(2\sqrt{BC}\right) I_{\omega-1}\left(2\sqrt{BC}\right) \\
&+ I_{\omega-1}^2\left(2\sqrt{BC}\right) + I_{\omega+1}^2\left(2\sqrt{BC}\right) + \left(\frac{C}{B}\right)^{-\omega} \\
&\times I_{\omega+1}\left(2\sqrt{BC}\right) I_{\omega-1}\left(2\sqrt{BC}\right) \Bigg\}^{1/2},
\end{aligned} \tag{30}$$

with the coefficients A, B, and C, given by equations (16–18). These longitudinal eigenvalues, however, lead to familiar mean-field results. At small values of the random field, the paramagnetic solution is unstable, while a ferromagnetic solution is stable. At large values of the random field, there is only a (stable) paramagnetic solution as in the case of spin glasses on the Bethe lattice (note that, at sufficiently large random fields, that is, for $H_R \to \infty$, we have $A, D \to 0$). The critical border separating these paramagnetic and ferromagnetic phase is of order cJ (see Tab. 1 for specific values).

We now turn to the eigenvalues associated with the transversal sector. For $\mu = 0$, and an eigenvector of the form

$$\begin{aligned}
\varphi_{\mu=0}\left(\frac{ix}{\beta}, \frac{iy}{\beta}\right) = &A_0 + B_{1+}\exp(ixJ) + B_{1-}\exp(-ixJ) \\
&+ B_{2+}\exp(iyJ) + B_{2-}\exp(-iyJ) + C_{++}\exp(ixJ + iyJ) \\
&+ C_{+-}\exp(ixJ - iyJ) \\
&+ C_{-+}\exp(-ixJ + iyJ) + C_{--}\exp(-ixJ - iyJ),
\end{aligned} \tag{31}$$

the problem is reduced to the calculation of the eigenvalues of a 9×9 matrix. The nine eigenvalues of this transversal sector are given by $\lambda_{T1} = 1/c$, associated with a constant eigenvector,

$$\lambda_{T2,T3} = \frac{1}{c}(1 - A), \tag{32}$$

$$\lambda_{T4,T5,T6} = \frac{1}{c}(1 - A) + D, \tag{33}$$

and

$$\lambda_{T7,T8,T9} = \frac{1}{c}(1 - A) - D, \tag{34}$$

which should be compared with equation (29) for the nontrivial eigenvalues of the longitudinal sector. In contrast to the spin-glass case, the eigenvalues of this transverse sector do not lead to any additional instability. According to a numerical analysis of these eigenvalues, the replica-symmetric paramagnetic solution remains stable for sufficiently large random fields and the ferromagnetic solution is stable in its region of existence (see Tab. 1).

Table 1

H_R/J	0	1	2	3	4	c
λpm	−0.1798	−0.0661	0.1030	0.2253	0.2919	3
λfm	0.2738	0.1157	− − −	− − −	− − −	3
λpm	−0.1876	−0.1159	0.0129	0.1212	0.1906	4
λfm	0.2302	0.1669	− − −	− − −	− − −	4
λpm	−0.1876	−0.1377	−0.0369	0.0578	0.1258	5
λfm	0.1930	0.1666	0.0819	− − −	− − −	5
λpm	−0.1847	−0.1476	−0.0667	0.0161	0.0808	6
λfm	0.1642	0.1529	0.1153	− − −	− − −	6

The analysis of the transversal sector with $\mu \neq 0$ can be carried out with same Ansatz,

$$\varphi_{\mu\neq 0}\left(\frac{ix}{\beta},\frac{iy}{\beta}\right) = \sum_{\theta\in\mathbf{Z}} \exp\left(\frac{1}{2} im\theta J\right)\varphi_{\theta,\mu=0}\left(\frac{ix}{\beta},\frac{iy}{\beta}\right). \tag{35}$$

Although the secular matrix becomes infinite, it is easy to see that the eigenvalues are still given by the same expressions of equations (32–34). Again, we conclude that the replica-symmetric solution is stable in the presence of sufficiently large random fields.

In Table 1, we list the numerical solutions for the smallest eigenvalue, given by equation (34), with $c = 3, 4, 5$, and 6, for the paramagnetic (pm) and ferromagnetic (fm) solutions. There is no simultaneous instability of both solutions and thus no indication of breaking of replica symmetry. Note that the dashes in this table correspond to the absence of a ferromagnetic solution (in which case the paramagnetic solution is stable).

4 Conclusions

We have investigated the stability of the replica-symmetric solutions of a random-field Ising ferromagnet on a lattice of finite mean connectivity. At low temperatures and for sufficiently large random fields, the analysis of the eigenvalues of the Hessian matrix associated with the variational free energy leads to stable replica-symmetric paramagnetic solutions (at smaller random fields, the replica-symmetric ferromagnetic solution is stable). The present calculations do not support the existence of a glassy phase, as suggested by earlier proposals [2–4]. However, a more detailed analysis of the phase diagram, in terms of field and temperature, still demands considerable work, including both analytical and refined numerical calculations. The assumption of a discrete distribution of effective fields, which was written as a sum of delta functions, may not capture the subtleties of the glassy behavior. As in the spin-glass case, we cannot rule out the existence of field-induced glassy and mixed ferromagnetic-glassy phases.

It is interesting to point out a connection with the recent work by Pastor and collaborators [3]. A truncation of the variational free energy, given by equation (4), leads to a "high-temperature approximation,"

$$f_{app} = \frac{1}{\beta}\lim_{n\to 0}\frac{1}{n}\left\{\frac{c}{2}\tanh(\beta J)\sum_{\alpha} m_{\alpha}^2 + \frac{c}{2}\left[\tanh(\beta J)\right]^2\sum_{\alpha<\beta} q_{\alpha\beta}^2 - \ln\left[\int_{-\infty}^{+\infty} dH p_H(H)\, Z_{app}\right]\right\}, \tag{36}$$

where

$$Z_{app} = \mathrm{Tr}_{\sigma}\exp\left\{c\tanh(\beta J)\sum_{\alpha}\sigma_{\alpha} + c\left[\tanh(\beta J)\right]^2\sum_{\alpha<\beta} q_{\alpha\beta}\sigma_{\alpha}\sigma_{\beta} + \beta H\sum_{\alpha=1}^{n}\sigma_{\alpha}\right\}. \tag{37}$$

The results of Pastor et al. [3] for the paramagnetic phase are recovered if we introduce the additional approximation $\tanh\beta J = \beta J + O\left[(\beta J)^3\right]$, and discard higher-order terms. In this approximation, for a Gaussian distribution of random fields with variance H_R, the replica-symmetric paramagnetic phase is unstable along the field axis.

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